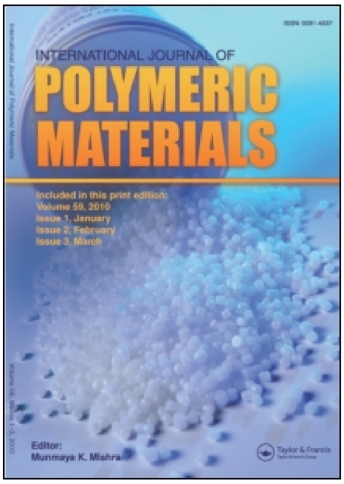

International Journal of Polymeric Materials
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713647664

Analysis on Composite Flow of Polymer Melts in Sandwich Sheeting Die

Woochun Hsu[ab]; J. S. Sheu[a]; C. Y. Hsieh[a]; Katsuhiko Ito[ac]
[a] Department of Chemical Engineering, National Cheng Kung University Tainan, Taiwan, Republic of China [b] Department of Chemistry, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong [c] Department of Mechanical Engineering, College of Engineering, Hosei University, Tokyo, Japan

To cite this Article Hsu, Woochun , Sheu, J. S. , Hsieh, C. Y. and Ito, Katsuhiko(1977) 'Analysis on Composite Flow of Polymer Melts in Sandwich Sheeting Die', International Journal of Polymeric Materials, 5: 4, 303 — 315
To link to this Article: DOI: 10.1080/00914037708075214
URL: http://dx.doi.org/10.1080/00914037708075214

Intern. J. Polymeric Mater., 1977, Vol. 5, pp. 303–315

Printed in Reading, England

Analysis on Composite Flow of Polymer Melts in Sandwich Sheeting Die

WOOCHUN HSU,† J. S. SHEU, C. Y. HSIEH and KATSUHIKO ITO‡

Department of Chemical Engineering, National Cheng Kung University Tainan, Taiwan, Republic of China

(*Received June 1975*)

A broken section method for analyzing the two-phase flow of polymer melts in a sandwich sheeting die under a pressure gradient, is presented. The method assumes that the laminar flows are isothermal and the incompressive materials are power-law fluids. Two polymer melts having different flow behaviors are extruded from two extruders respectively into a coaxial manifold and simultaneously, flow into slits perpendicular to the manifold axis, and then contact with each other in the final slit of the sheeting die for laminating the sandwich sheet. The immiscible interface of these adjacent flows in this final slit is determined by a method of variation. Simultaneous equations in many broken sections are analyzed by computerized successive approximation. Uniformity is defined for both core and skin layers.

INTRODUCTION

Processing of sandwiched sheet of plastics by extrusion has become an important practice in the polymer industry. At this stage of the art, the dies are designed based mainly on experience. Analytical work on polymer melt flow in slit die, a simpler problem than the present one, has been reported by a number of authors.[1-3] This work is intended to analyze polymer melt flow in a sandwich sheet die, so that the contributions of the parameters affecting the uniformity of sandwich sheet product may be inferred. The broken section method, which has been proved to be a useful tool in polymer flow analysis,[3,4] is employed to analyze this problem.

---

† Present Address: Department of Chemistry, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong.

‡ To whom all communications should be addressed. Present Address: Department of Mechanical Engineering, College of Engineering, Hosei University, Kajinocho, Koganei, Tokyo, Japan.

ANALYSIS

The physical system

Figure 1 shows the schematic diagram of an end-feeding sandwich sheeting die. Polymer melts are delivered to the inner manifold from left to right by the inner extruder and to the outer manifold from right to left by the outer extruder. Along the manifold, polymer melts passing through the slits (Figure 2) meet each other and then extrude out. In the manifold, the two polymer melts having different flow behaviors flow independently of each other. In this analysis, it was assumed that the polymer melts are incompressible, immiscible and are power-law fluids, and the flows are steady, laminar and isothermal. The die is divided into N sections, the length S of each section is:

$$S = \frac{L}{N} \tag{1}$$

In the following, the manifold and slit flow in the inner and outer parts of the die were considered first, the results were then applied to analyze the composite flow in a sandwich sheeting die, and the mathematical expression for uniformity was then derived.

Separated inner manifold and slit flow

Consider an arbitrary cross section in the manifold; $P_{i,j-1}$ is the pressure at the start of the jth section, and $P_{i,j}$ is that at the end. Hence $(P_{i,j-1} - P_{i,j})$ is the pressure drop across this section. Similarly, $q_{i,j-1}$ and $q_{i,j}$ are the volumetric flow rates at the start and the end of this section, respectively. The average flow rate in the manifold is the mean of $q_{i,j-1}$ and $q_{i,j}$, i.e., $\frac{1}{2}(q_{i,j-1} + q_{i,j})$. The relationship between the flow rate and the pressure drop for the jth section of the inner manifold flow is as follows:[3,5]

$$\tfrac{1}{2}(q_{i,j-1} + q_{i,j}) = \phi_i M_{i,j}(p_{i,j-1} - p_{i,j})^{1/n_i} \tag{2}$$

where

$$\phi_i \equiv \left(\frac{P_{i,0}}{\eta_i^0 \cdot \dot{\gamma}_i^0}\right)^{1/n_i} \tag{3}$$

$$p_{i,j} \equiv (P_{i,j}/P_{i,0}) \tag{4}$$

$$M_{i,j} = \left(\frac{n_i \pi R_{i,j}^3 \dot{\gamma}_i^0}{3n_i + 1}\right)\left(\frac{R_{i,j}}{2S}\right)^{1/n_i} \tag{5}$$

and n_i: flow-index of inner polymer

$P_{i,0}$: inlet pressure of inner flow

$p_{i,j}$: reduced pressure of inner flow at the jth section.

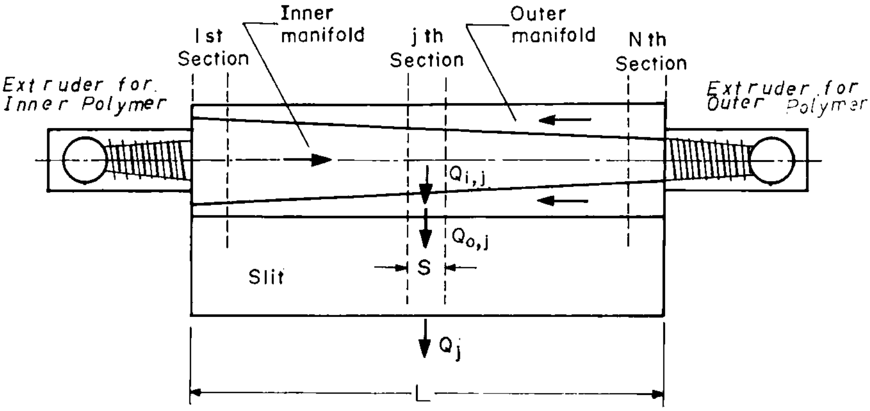

FIGURE 1 General view of end-feeding sandwich sheet processing.

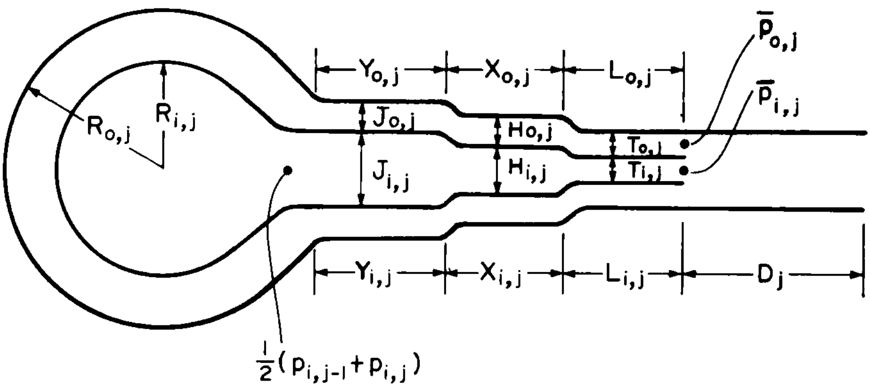

FIGURE 2 Cross section of the jth section of sandwich sheeting die, showing principal dimensions and intermediate pressures.

From material balance, the following equation is obtained

$$q_{i,j-1} = \sum_{m=j}^{N} Q_{i,m} \tag{6}$$

The flow in the slit is essentially flow between parallel plates, the relationship between the flow rate and pressure drop is as follows:[3,6]

$$Q_{i,j} = \phi_i K_{i,j}\left(\frac{p_{i,j-1} + p_{i,j}}{2} - \bar{p}_{i,j}\right)^{1/n_i} \tag{7}$$

where $\bar{P}_{i,j}$: the intermediate pressure of inner flow at the joining section of both flows.

$$\frac{1}{K_{i,j}} = \left\{\frac{1}{(A_{i,j})^{n_i}} + \frac{1}{(B_{i,j})^{n_i}} + \frac{1}{(C_{i,j})^{n_i}}\right\}^{1/n_i} \tag{8}$$

$$\left.\begin{aligned} A_{i,j} &= \frac{SJ_{i,j}^2 n_i \dot{\gamma}_i^0}{4n_i + 2}\left(\frac{J_{i,j}}{2Y_{i,j}}\right)^{1/n_i} \\ B_{i,j} &= \frac{SH_{i,j}^2 n_i \dot{\gamma}_i^0}{4n_i + 2}\left(\frac{H_{i,j}}{2X_{i,j}}\right)^{1/n_i} \\ C_{i,j} &= \frac{ST_{i,j}^2 n_i \dot{\gamma}_i^0}{4n_i + 2}\left(\frac{T_{i,j}}{2L_{i,j}}\right)^{1/n_i} \end{aligned}\right\} \tag{9}$$

Separated outer manifold and slit flow

Consider outer flow in the manifold as essentially flow in an annulus, the equations relating flow rate and pressure drops are as follows:[3,7,8]

$$\tfrac{1}{2}(q_{0,j-1} + q_{0,j}) = \phi_0 M_{0,j}(p_{0,j} - p_{0,j-1})^{1/n_0} \tag{10}$$

where $$\phi_0 = \left(\frac{P_{0,N}}{\eta_0^0 \dot{\gamma}_0^0}\right)^{1/n_0} \tag{11}$$

$$p_{0,j} = (P_{0,j}/P_{0,N}) \tag{12}$$

$$M_{0,j} = \left(\frac{n_0 \pi R_{0,j}^3 \dot{\gamma}_0^0}{2n_0 + 1}\right)\left(\frac{R_{0,j}}{2S}\right)^{1/n_0}\left(\frac{\beta_j - 1}{\beta_j}\right)^{(2n_0+1)/n_0} F(n_0, \beta_j) \tag{13}$$

$$j = (R_{0,j}/R_{i,j}) \tag{14}$$

n_0: flow index of outer polymer

$P_{0,N}$: inlet pressure of outer flow

$p_{0,j}$: reduced pressure of outer flow at the jth section

$F(n_0, \beta_j)$: a function depends only upon n_0 and β_j, whose values have been obtained by Fredrickson and Bird.[8]

From mass balance, Eq. (15) is obtained:

$$q_{0,j-1} = \sum_{m=1}^{j-1} Q_{0,m} \quad \text{or} \quad q_{0,j} = \sum_{m=1}^{j} Q_{0,m} \tag{15}$$

In the slit, there is also a flow between parallel plates. Further it is assumed that the outer slits in both sides are arranged symmetrically to the slit z-axis, as shown in Figure 3: $2T_{f,i} = 2T_{0,i} + T_{i,i}$.

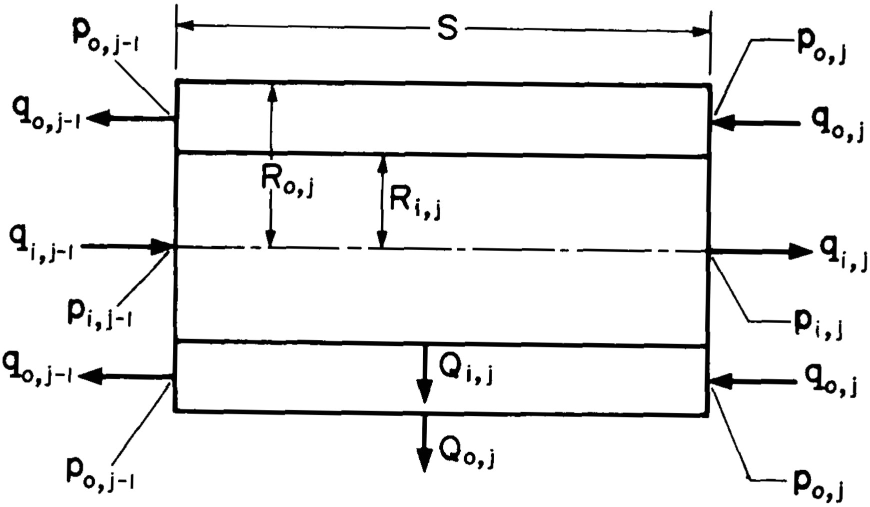

FIGURE 3 Cross section of two manifolds at the jth section, showing flow rates and pressures.

$$Q_{0,j} = 2\phi_0 K_{0,j}\left(\frac{p_{0,j-1} + p_{0,j}}{2} - \bar{p}_{0,j}\right)^{1/n_0} \tag{16}$$

where

$$\frac{1}{K_{0,j}} = \left\{\frac{1}{(A_{0,j})^{n_0}} + \frac{1}{(B_{0,j})^{n_0}} + \frac{1}{(C_{0,j})^{n_0}}\right\}^{1/n_0} \tag{17}$$

$$\left.\begin{aligned} A_{0,j} &= \frac{SJ_{0,j}^2 n_0 \dot{\gamma}_0^0}{4n_0 + 2}\left(\frac{J_{0,j}}{2Y_{0,j}}\right)^{1/n_0} \\ B_{0,j} &= \frac{SH_{0,j}^2 n_0 \dot{\gamma}_0^0}{4n_0 + 2}\left(\frac{H_{0,j}}{2X_{0,j}}\right)^{1/n_0} \\ C_{0,j} &= \frac{ST_{0,j}^2 n_0 \dot{\gamma}_0^0}{4n_0 + 2}\left(\frac{T_{0,j}}{2L_{0,j}}\right)^{1/n_0} \end{aligned}\right\} \tag{18}$$

$\bar{P}_{0,j}$: the intermediate pressure of outer flow at the joining section of both flows.

Composite flow in sandwich sheeting die

As the inner and outer polymer melts are brought into contact with each other in the final part of the die lip, i.e. in the sandwich sheeting die, as shown in Figure 4.

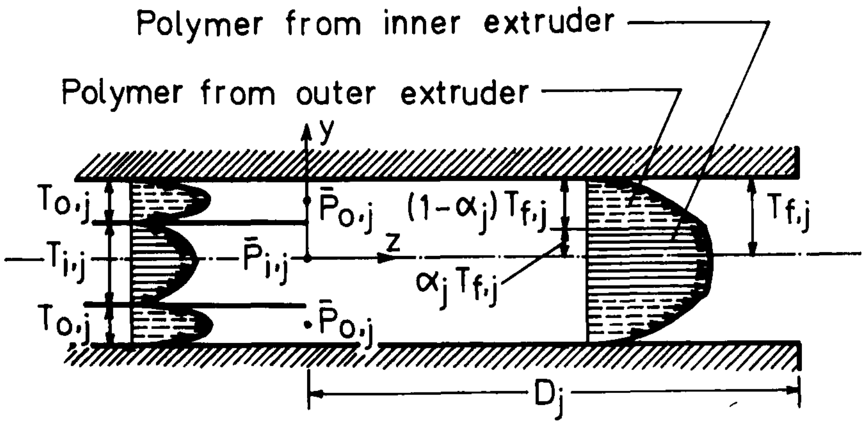

FIGURE 4 Two-phase flow behavior of final slit in sandwich sheeting die.

For simplicity of mathematical treatment, the immiscibility of those two melts was assumed, the entrance effects were omitted, and the pressure gradient in the region of two phase flow is assumed to be:

$$\frac{\partial p}{\partial z} = \frac{T_{i,j}\bar{P}_{i,j} + 2T_{0,j}\bar{P}_{0,j}}{D_j(T_{i,j} + 2T_{0,j})} \tag{19}$$

which is constant across the two-phase flow region.

The steady-state velocity profiles for the inner and outer melts at the upper part of this region are (6):

$$v_{i,j} = -(\dot{\gamma}_i^0)^{(n_i-1)/n_i}\left[\frac{1}{\eta_i^0}\left(\frac{\partial p}{\partial z}\right)\right]^{1/n_i}\frac{n_i}{1+n_i}y^{(n_i+1)/n_i} + C_i \tag{20}$$

$$v_{0,j} = -(\dot{\gamma}_0^0)^{(n_0-1)/n_0}\left[\frac{1}{\eta_0^0}\left(\frac{\partial p}{\partial z}\right)\right]^{1/n_0}\frac{n_0}{1+n_0}y^{(n_0+1)/n_0} + C_0 \tag{21}$$

Since two phase composite flows in the sandwich sheeting die are symmetrical to the slit z-axis, as shown in Figure 4, the boundary conditions are as follows.

$$v_{i,j} = v_{0,j}, \text{ at } y = \alpha_j T_{f,j} \quad . \quad \text{(Determination of } C_i)$$

$$v_{0,j} = 0, \quad \text{at } y = T_{f,j} \quad . \quad \text{(Determination of } C_0)$$

where α_j: coefficient for determining the position of interface of two phase composite flows in the jth section.

The complete velocity profiles are then:

$$v_{i,j} = G_{i,j}[(\alpha_j T_{f,j})^{(n_i+1)/n_i} - y^{(n_i+1)/n_i}] + G_{0,j}[(T_{f,j})^{(n_0+1)/n_0} - (\alpha_j T_{f,j})^{(n_0+1)/n_0}] \quad (22)$$

$$v_{0,j} = G_{0,j}[(T_{f,j})^{(n_0+1)/n_0} - y^{(n_0+1)/n_0}] \quad (23)$$

where

$$\left.\begin{aligned} G_{i,j} &\equiv (\dot{\gamma}_i^0)^{(n_i-1)/n_i}\left[\frac{1}{\eta_i^0}\left(\frac{\partial p}{\partial z}\right)^{1/n_i}\right]\cdot\left(\frac{n_i}{1+n_i}\right) \\ G_{0,j} &= (\dot{\gamma}_0^0)^{(n_0-1)/n_0}\left[\frac{1}{\eta_0^0}\left(\frac{\partial p}{\partial z}\right)^{1/n_0}\right]\cdot\left(\frac{n_0}{1+n_0}\right) \end{aligned}\right\} \quad (24)$$

The volumetric flow rates in the z-direction are given by the integral,

$$\begin{aligned} Q_{i,j} &= 2S\int_0^{\alpha_j T_{f,j}} v_{i,j}\,dy \\ &= 2SG_{i,j}\left[(\alpha_j T_{f,j})^{(2n_i+1)/n_i} - \frac{n_i}{2n_i+1}\cdot(\alpha_j T_{f,j})^{(2n_i+1)/n_i}\right] \\ &\quad + 2SG_{0,j}[\alpha_j(T_{f,j})^{(2n_0+1)/n_0} - (\alpha_j T_{f,j})^{(2n_0+1)/n_0}] \end{aligned} \quad (25)$$

$$\begin{aligned} Q_{0,j} &= 2S\int_{\alpha_j T_{f,j}}^{T_{f,j}} v_{0,j}\,dy \\ &= 2SG_{0,j}\Big[(T_{f,j})^{(2n_0+1)/n_0} - \alpha_j(T_{f,j})^{(2n_0+1)/n_0} \\ &\quad - \frac{n_0}{2n_0+1}(T_{f,j})^{(2n_0+1)/n_0} + \frac{n_0}{2n_0+1}(\alpha_j T_{f,j})^{(2n_0+1)/n_0}\Big] \end{aligned} \quad (26)$$

where $\alpha_j T_{f,j}$ is the distance between the central line and the line of interface generated in the sandwich flow.

i) In the case when $\bar{p}_{i,j} \neq \bar{p}_{0,j}$

In general $\bar{p}_{i,j}$ is not exactly equal to $\bar{p}_{0,j}$. For the case where the inlet pressures ($P_{i,0}$ and $P_{0,N}$) and the geometrical configuration of the die are known, there are $9N$ unknown quantities of $p_{i,j}s$, $\bar{p}_{i,j}s$, $p_{0,j}s$, $\bar{p}_{0,j}s$, $q_{i,j}s$, $Q_{i,j}s$, $q_{0,j}s$, $Q_{0,j}s$ and $\alpha_j s$. Because independent equations generated from Eqs. (2), (6), (7), (10), (15), (16), (25) and (26) are $8N$, the problem is not completely formulated. Hence α_j is determined approximately by Schechter's variation method by setting the Johnson's energy function J_j in the final sandwich die slit of the jth section, to be minimized (9–12):

i.e. $$\partial J_j/\partial \alpha_j = 0$$

The integral expression J_j for power-law fluid in this case is

$$\frac{J_j}{2} = \int_0^{\alpha_j T_{f,j}} \left\{ \frac{\tau_i^0 \cdot \dot{\gamma}_i^0}{n_i + 1} \left[\left(\frac{1}{\dot{\gamma}_i^0} \frac{\partial v_{i,j}}{\partial y} \right)^2 \right]^{(n_i+1)/2} + \left(\frac{\partial p}{\partial z} \right) v_{i,j} \right\} dy$$

$$+ \int_{\alpha_j T_{f,j}}^{T_{f,j}} \left\{ \frac{\tau_0^0 \dot{\gamma}_0^0}{n_0 + 1} \left[\left(\frac{1}{\dot{\gamma}_0^0} \frac{\partial v_{0,j}}{\partial y} \right)^2 \right]^{(n_0+1)/2} + \left(\frac{\partial p}{\partial z} \right) v_{0,j} \right\} dy \quad (27)$$

The final equation for α_j obtained this way is

$$\alpha_j = \left\{ \frac{(\dot{\gamma}_0^0)^{(n_0-1)/n_0} \left(\frac{1}{\eta_0^0} \right)^{1/n_0} \left(\frac{n_0 + 2}{n_0 + 1} \right)}{(\dot{\gamma}_i^0)^{(n_i-1)/n_i} \left(\frac{1}{\eta_i^0} \right)^{1/n_i} \left(\frac{n_i + 2}{n_i + 1} \right)} \right\}^{n_i n_0 / n_0 - n_i} \left[\frac{2D_j}{T_{i,j}\bar{p}_{i,j} + 2T_{0,j}\bar{p}_{0,j}} \right] \quad (28)$$

Thus the $9N$ unknown quantities can be solved by simultaneous equations (2), (6), (7), (10), (15), (16), (25), (26) and (28). Eliminating of $4N$ unknown quantities: $q_{i,j}s$, $Q_{i,j}s$, $q_{0,j}s$ and $Q_{0,j}s$ yields the following five sets of working equations:

$$\alpha_j = \frac{F_j}{T_{i,j}\bar{p}_{i,j} + 2T_{0,j}\bar{p}_{0,j}} \quad (29)$$

$$p_{i,j} = p_{i,j-1} - \left\{ \frac{K_{i,j}}{2M_{i,j}} \left(\frac{p_{i,j-1} + p_{i,j}}{2} - \bar{p}_{i,j} \right)^{1/n_i} + \frac{1}{M_{i,j}} \sum_{m-j+1}^{N} K_{i,m} \left(\frac{p_{i,m-1} + p_{i,m}}{2} \right) - \bar{p}_{i,m} \right)^{1/n_i} \Bigg\}^{n_i} \quad (30)$$

$$\bar{p}_{i,j} = \frac{p_{i,j-1} + p_{i,j}}{2} - \{E_{i,j}(T_{i,j}\bar{p}_{i,j} + 2T_{0,j}\bar{p}_{0,j})^{1/n_i} + E_{0,j}(T_{i,j}\bar{p}_{i,j} + 2T_{0,j}\bar{p}_{0,j})^{1/n_0}\}^{n_i} \quad (31)$$

$$p_{0,j} = p_{0,j-1} + \left\{ \frac{K_{0,j}}{M_{0,j}} \left(\frac{p_{0,j-1} + p_{0,j}}{2} - \bar{p}_{0,j} \right)^{1/n_0} + \frac{1}{M_{0,j}} \sum_{m=1}^{j-1} K_{0,m} \left(\frac{p_{0,m-1} + p_{0,m}}{2} - \bar{p}_{0,m} \right)^{1/n_0} \right\}^{n_0} \quad (32)$$

$$\bar{p}_{0,j} = \frac{p_{0,j-1} + p_{0,j}}{2} - N_{0,j}(T_{i,j}\bar{p}_{i,j} + 2T_{0,j}\bar{p}_{0,j}) \quad (33)$$

where

$$\left.\begin{aligned}
F_j &= 2D_j\left\{\frac{(\dot{\gamma}_0^0)^{(n_0-1)/n_0}\left(\dfrac{1}{\eta_0^0}\right)^{1/n_0}\left(\dfrac{n_0+2}{n_0+1}\right)}{(\dot{\gamma}_i^0)^{(n_i-1)/n_i}\left(\dfrac{1}{\eta_0^0}\right)^{1/n_i}\left(\dfrac{n_i+2}{n_i+1}\right)}\right\}^{n_i n_0/n_0-n_i}\\
E_{i,j} &= \left(\frac{2ST_{f,j}^2}{\phi_i K_{i,j}}\right)(\dot{\gamma}_i^0)^{(n_i-1)/n_i}\left(\frac{n_i}{2n_i+1}\right)\left(\frac{1}{2\eta_i^0 D_j}\right)^{1/n_i}(\alpha_j)^{2n_i+1/n_i}\\
E_{0,j} &= \left(\frac{2ST_{f,j}^2}{\phi_i K_{i,j}}\right)(\dot{\gamma}_0^0)^{(n_0-1)/n_0}\left(\frac{n_0}{n_0+1}\right)\left(\frac{1}{2\eta_0^0 D_j}\right)^{1/n_0}(\alpha_j-\alpha_j^{2n_0+1/n_0})\\
N_{0,j} &= \left\{\left(\frac{ST_{f,j}^2}{\phi_0 K_{0,j}}\right)(\dot{\gamma}_0^0)^{(n_0-1)/n_0}\left(\frac{n_0}{n_0+1}\right)\left(\frac{1}{2\eta_0^0 D_j}\right)^{1/n_0}\left[\left(\frac{n_0+1}{2n_0+1}\right)-\alpha_j\right.\right.\\
&\qquad\left.\left.+\left(\frac{n_0}{2n_0+1}\right)\alpha_j^{2n_0+1/n_0}\right]\right\}^{n_0}
\end{aligned}\right\}\quad(34)$$

ii) In the case when $\bar{p}_{i,j}=\bar{p}_{0,j}\ (\equiv\bar{p}_j)$

It is our industrial practice that D_j is to be small for reducing the flow resistance in sandwich sheeting die, unless the mutual adhesion between two melts is insufficient. From this viewpoint, a die design where manifold and slit dimensions are such that $\bar{p}_{i,j}$, is so different from $\bar{p}_{0,j}$, is not desirable. In the case when $\bar{p}_{i,j}=\bar{p}_{0,j}(\equiv\bar{p}_j)$, the simultaneous equations of (2), (6), (7), (10), (15), (16), (25) and (26) contain $8N$ unknown quantities of $p_{i,j}s$, $p_{0,j}s$, $\bar{p}_js$, $q_{i,j}s$, $Q_{i,j}s$, $q_{0,j}s$, $Q_{0,j}s$ and α_js. Hence the problem is completely formulated without the application of variational approach for α_j. In the same way in the case of (i), the elimination of $4N$ quantities: $q_{i,j}s$, $Q_{i,j}s$, $q_{0,j}s$ and $Q_{0,j}s$ yields the following four sets of working equations:

$$p_{i,j}=p_{i,j-1}-\left\{\frac{K_{i,j}}{2M_{i,j}}\left(\frac{p_{i,j-1}+p_{i,j}}{2}-\bar{p}_j\right)^{1/n_i}+\frac{1}{M_{i,j}}\sum_{m=j+1}^{N}K_{i,m}\left(\frac{p_{i,m-1}+p_{i,m}}{2}-\bar{p}_m\right)^{1/n_i}\right\}^{n_i}\quad(35)$$

$$\bar{p}_j=\left(\frac{p_{i,j-1}+p_{i,j}}{2}\right)-\{S_{i,j}(\alpha_j)^{(2n_i+1)/n_i}(\bar{p}_j)^{1/n_i}+S_{0,j}(\alpha_j-\alpha_j^{(2n_0+1)/n_0})(\bar{p}_j)^{1/n_0}\}^{n_i}\quad(36)$$

$$p_{0,j}=p_{0,j-1}+\left\{\frac{K_{0,j}}{M_{0,j}}\left(\frac{p_{0,j-1}+p_{0,j}}{2}-\bar{p}_j\right)^{1/n_0}+\frac{2}{M_{0,j}}\sum_{m=1}^{j-1}K_{0,m}\left(\frac{p_{0,m-1}+p_{0,m}}{2}-\bar{p}_m\right)^{1/n_0}\right\}^{n_0}\quad(37)$$

$$\alpha_j = \left(\frac{n_0 + 1}{2n_0 + 1} + \frac{n_0}{2n_0 + 1}\alpha_j^{2n_0+1/n_0}\right) - V_{0,j}(\bar{p}_j)^{-1/n_0}\left(\frac{p_{0,j-1} + p_{0,j}}{2} - \bar{p}_j\right)^{1/n_0} \quad (38)$$

where

$$\left.\begin{aligned} S_{i,j} &\equiv \left(\frac{2S}{\phi_j K_{i,j}}\right)(\dot{\gamma}_i^0)^{(n_i-1)/n_i}\left(\frac{n_i}{2n_i+1}\right)\left(\frac{1}{D_j \eta_i^0}\right)^{1/n_i}(T_{f,j})^{(2n_i+1)/n_i} \\ S_{0,j} &\equiv \left(\frac{2S}{\phi_i K_{i,j}}\right)(\dot{\gamma}_0^0)^{(n_0-1)/n_0}\left(\frac{n_0}{n_0+1}\right)\left(\frac{1}{D_j \eta_0^0}\right)^{1/n_0}(T_{fj})^{(2n_0+1)/n_0} \\ \frac{1}{V_{0,j}} &\equiv \left(\frac{S}{\phi_0 K_{0,j}}\right)(\dot{\gamma}_0^0)^{(n_0-1)/n_0}\left(\frac{n_0}{n_0+1}\right)\left(\frac{1}{D_j \eta_0^0}\right)^{1/n_0}(T_{fj})^{(2n_0+1)/n_0} \end{aligned}\right\} \quad (39)$$

All of these simultaneous equations may be solved approximately by the method of iteration.

Uniformity

For the inner extrudate, the relative deviation from the mean of the flow from the jth section: $\Delta_{i,j}$ is given by

$$\Delta_{i,j} = \frac{Q_{i,j} - (Q_{i,0}/N)}{(Q_{i,0}/N)} = N\left(\frac{Q_{i,j}}{Q_{i,0}}\right) - 1$$

i.e.

$$\Delta_{i,j} = N\frac{K_{i,j}\left(\frac{p_{i,j-1} + p_{i,j}}{2} - \bar{p}_{i,j}\right)^{1/n_i}}{\sum_{j=1}^{N} K_{i,j}\left(\frac{p_{i,j-1} + p_{i,j}}{2} - \bar{p}_{i,j}\right)^{1/n_i}} - 1 \quad (40)$$

where $Q_{i,0} = \sum_{m=1}^{N} Q_{i,m}$: total volumetric flow rate of inner extrudate. Sometimes it is convenient to characterize the flow uniformity by a single number rather than the graphical plotting of $\Delta_{i,j}$. For this purpose a uniformity function, U_i, is defined in terms of the standard deviation of $\Delta_{i,j}$ by the equation

$$U_i = 1 - \sqrt{\frac{1}{N}\sum_{j=1}^{N}(\Delta_{i,j})^2} \quad (41)$$

Similarly, the relative deviation $\Delta_{0,}$ and the uniformity U_0 for the outer extrudate are given by

$$\Delta_{0,j} = \frac{Q_{0,j} - (Q_{0,0}/N)}{(Q_{0,0}/N)} = N\left(\frac{Q_{0,j}}{Q_{0,0}}\right) - 1$$

i.e.

$$\Delta_{0,j} = N \frac{K_{0,j}\left(\dfrac{p_{0,j-1} + p_{0,j}}{2} - \bar{p}_{0,j}\right)^{1/n_0}}{\sum\limits_{=1}^{N} K_{0,j}\left(\dfrac{p_{0,j-1} + p_{0,j}}{2} - \bar{p}_{0,j}\right)^{1/n_0}} - 1 \tag{42}$$

where $Q_{0,0} = \sum\limits_{j=1}^{N} Q_{0,j}$: total volumetric flow rate of outer extrudate.

$$U_0 = 1 - \sqrt{\frac{1}{N} \sum_{j=1}^{N} (\Delta_{0,j})^2} \tag{43}$$

The relative deviation Δ_j and the uniformity U for the overall flow of both the polymer melts are similarly given by

$$\Delta_j = \frac{Q_j - (Q_0/N)}{(Q_0/N)} = N\left(\frac{Q_j}{Q_0}\right) - 1 \tag{44}$$

$$U = 1 - \sqrt{\frac{1}{N} \sum_{j=1}^{N} (\Delta_j)^2} \tag{45}$$

where Q_j: volumetric flow rate from the jth section.

$Q_0 = Q_{i,0} + Q_{0,0} = \sum\limits_{j=1}^{N} Q_j$: total volumetric flow rate of both inner and outer extrudates.

U is a function of both flow indices: n_i and n_0. For all polymer melts having various flow indices from zero to unity, hence, an average uniformity $\bar{U}$ of sandwich sheeting die is defined by the equation

$$\bar{U} = \int_0^1 \int_0^1 U \, dn_i dn_0 \tag{46}$$

DISCUSSION

Equations (41) and (43) give the uniformity of a sandwich sheeting die, it is dependent on the K's, p_i's, p_0's, and the flow indices n_i and n_0. The K's and the p's are functions of the die configuration and the flow indices. The flow indices of the polymer melt are the most important factor influencing the uniformity of the sandwich sheet.

Acknowledgement

The authors wish to express their appreciation to the Chinese National Research Council for their financial support during the period of Professor Ito's stay in the Republic of China from February 1974 to May 1974. Professor Ito spent these months at the Cheng Kung University as Visiting Professor of Chemical Engineering. Further the authors are gratefully indebted to Professors T. T. Lai, M. K. Lu, J. H. Wang and the staff of Chemical Engineering, National Cheng Kung University, for their thoughtfulness during the period of this study.

List of symbols

$H_{i,j}$, $J_{i,j}$, $L_{i,j}$, $R_{i,j}$, $T_{i,j}$, $X_{i,j}$ and $Y_{i,j}$ = Dimensions for the inner polymer melt at the jth section of the die, as shown in Figure 2.

$H_{0,j}$, $J_{0,j}$, $L_{0,j}$, $R_{0,j}$, $T_{0,j}$, $X_{0,j}$ and $Y_{0,j}$ = Dimensions for the outer polymer melt at the jth section of the die, as shown in Figure 2.

$A_{i,j}$, $B_{i,j}$, $C_{i,j}$ and $K_{i,j}$ = Geometric parameters for inner slit flow through the jth section as defined by Eqs. (8) and (9).

$A_{0,j}$, $B_{0,j}$, $C_{0,j}$ and $K_{0,j}$ = Geometric parameters for outer slit flow through the jth section as defined by Eqs. (17) and (18).

D_j = Die length of the final die lip at the jth section, as shown in Figure 4.

$E_{i,j}$, $E_{0,j}$ = Variables corresponding to the inner and outer flow, respectively, at the jth section, as defined in Eq. (34).

F_j = A constant of jth section, as shown in Eq. (34).

$G_{i,j}$, $G_{0,j}$ = Constants for inner and outer melt, respectively, as defined in Eq. (24).

J_j = Johnson's energy function, as defined in Eq. (27).

L = Length of sandwich sheeting die.

$M_{i,j}$, $M_{0,j}$ = Geometric parameters for manifold flow through the jth section as defined in Eqs. (5) and (13), respectively.

N = Number of sections.

$N_{0,j}$ = Variable as defined in Eq. (34).

$P_{i,0}$, $P_{0,N}$ = Inlet pressures for inner and outer melt, respectively.

$P_{i,j}$, $P_{0,j}$ = Pressures at the far end of jth section.

$Q_{i,j}$, $Q_{0,j}$ = Volumetric slit flow rate from the jth section for inner and outer melt, respectively.

$q_{i,0}$, $q_{0,N}$ = Initial flow rates of inner and outer melt, respectively.

Q_0 = Total extrudate from die.

S = Length of each section.

$S_{i,j}$, $S_{0,j}$ = Constants as defined in Eq. (39).

$T_{f,j}$ = Half of the height of the die lip, as shown in Figure 4.

U = Overall uniformity as defined by Eq. (45).

U_i = Uniformity for the inner extrudate defined by Eq. (41).

$V_{i,j}$, $V_{0,j}$ = Velocity profiles corresponding to the inner and outer flow, respectively, at the jth section.

n_i, n_0 = Flow indices of inner and outer polymer melts which obey the power law.

$p_{i,j}$, $p_{0,j}$ = Reduced pressure defined by Eqs. (4) and (12) for inner and outer melt, respectively.

$q_{i,j}$, $q_{0,j}$ = Manifold flow rates from jth section.

y, z = Variables of direction as shown in Figure 4.

η = Viscosity.

η_0^0, η_i^0 = Standard-state viscosity of outer and inner melt, respectively.

$\dot{\gamma}$ = Shear rate.

$\dot{\gamma}_i^0$, $\dot{\gamma}_0^0$ = Standard-state shear rate for inner and outer melt, respectively.

τ_i^0, τ_0^0 = Standard shear stress for inner and outer melt, respectively, which can be seen as $\tau_i^0 = \eta_i^0 \dot{\gamma}_i^0$, and $\tau_0^0 = \eta_0^0 \dot{\gamma}_0^0$.

ϕ_i, ϕ_0 = Pressure parameters for inner and outer flow, respectively, as defined by Eqs. (3) and (11).

Δ_j = Relative flow deviation from the mean of total flow from the jth section, as defined by Eq. (44).

$\Delta_{i,j}$, $\Delta_{0,j}$ = Relative flow deviations for inner and outer extrudate, respectively, as defined by Eqs. (40) and (42).

References

1. J. F. Carley, *J. Appl. Phys.* **25,** 1118 (1954); *Modern Plastics* **33,** 127 (August 1956).
2. D. J. Weeks, *British Plastics*, p. 201 (May 1958).
3. J. M. McKelvey and K. Ito, *Polym. Eng. Sci.* **11,** 258 (1971).
4. K. Ito, M. Tsutsui and M. Kasajima, *App. Poly. Symp.* **20,** 99 (1973).
5. J. M. McKelvey, *Polymer Processing*, p. 69, John Wiley (1962).
6. Ibid., p. 102.
7. Ibid., p. 110.
8. A. G. Fredrickson and R. B. Bird, *Ind. Eng. Chem.* **50,** 347 (1958).
9. R. S. Schechter, *A.I.Ch. E. Journal* **7,** 445 (1961).
10. R. S. Schechter, *The Variational Method in Engineering*, Ch. 4, McGraw-Hill (1967).
11. D. L. MacLean, *Trans. Soc. Rheol.* **17,** 385 (1973).
12. Y. Tomita, *Bull. of JSME* **2,** 469 (1959).